BME120/220 Sensory Motor Systems
Solution to PBL # 1: Design of a FES-Driven Bicycle for a Person with Spinal Cord Injury

Background: Spinal Cord Injury and FES
About 10,000 people experience a spinal cord injury (SCI) in the U.S. each year, and over 200,000 people with a SCI are alive in the U.S. The leading cause of SCI is motor vehicle accidents (44%), followed by violence (24%) and falls (22%). About half of injuries are incomplete, meaning some motor or sensory function is preserved below the level of the lesion. About half of people with SCI are quadraplegic (arms and legs paralyzed), and about half are paraplegic (legs paralyzed). As defined by the Cleveland FES Center: “FES is a rehabilitation technique where low-level electrical current is applied to an individual with a disability to enhance that person’s ability to function and live independently. FES usually refers to applications which provide long term assistance or replacement of a missing or impaired body function.” Example uses of FES are for strengthening of muscles, control of the bladder, control of hand grasp, walking, and standing.

2. Muscle Selection and Stimulation Pattern
A four-channel stimulation system allows for the generation of four independent stimulation patterns. Thus, four muscle groups could be stimulated with different patterns. A good approach is to stimulate two muscle groups in each leg. A logical choice is to use one channel to stimulate the three hamstrings of one leg (semimembranosus, semitendinosus, biceps femoris) in order to flex the knee during the up-stroke, and to use a second channel to stimulate the three superficial vastii muscles (vastus lateralis, vastus intermedius, vastus medialis) to extend the knee during the down-stroke. The other two channels would be used for the other leg. Muscle excitation should be controlled by measuring crank position and triggering the proper muscles at the proper phase of the cycling motion, based on experimental measurements of muscle activation patterns (i.e. EMG) in unimpaired riders. Crank position could be measured with a potentiometer or optical encoder. Pairs of stimulation electrodes could be taped to the skin above each muscle at the muscle’s motor point, defined as the point at which the nerve enters the muscle, which has the lowest threshold for excitation of an action potential. Care should be taken in providing an appropriate stimulus intensity (typical < 120 mamp) and frequency (typical 20 Hz square wave, with the pulse width modulated up to a duration of 700 microseconds to increase recruitment) in order to minimize damage to the skin and fatigue. Rotating stimulation among muscle groups may also help reduce fatigue. The user could control the intensity of stimulation by changing the pulse width of the stimulus train.

3. Gear Shifting Algorithm
From muscle mechanics experiments it is known that there exists an optimal muscle velocity $V_{\text{opt}}$ and muscle force $F_{\text{opt}}$ at which muscle power output is maximized. Maximizing power output is desirable because it means for a given force level (i.e. determined by wind-resistance, friction, hills) and total time riding the bicycle, the customer will travel the maximum distance possible. It is possible to show that for isolated muscle, $V_{\text{opt}} = 0.3V_{\text{max}}$ and $F_{\text{opt}} = 0.3F_{\text{max}}$ (see attached derivation), where $V_{\text{max}}$ is the maximal velocity (zero load) and $F_{\text{max}}$ is the maximal force (isometric).
Again, it would be expected that all synergist muscles working together during pedaling would exhibit an optimal power output at some angular velocity and force. For this problem, you are told to assume that the Hill curve describes the torque/angular velocity relationship for pedaling at a constant level of muscle activation. Thus, it is logical to try to design the bicycle so the user pedals at the optimal power point on the Hill Torque-Angular Velocity curve (i.e. $\omega_{\text{opt}}$, $\tau_{\text{opt}}$). To allow the customer to pedal at the optimal power point, an appropriate gear ratio is needed.

One way to select the appropriate gear ratio is based on feedback of the pedal crank velocity $\omega$, measured with a tachometer, for example. The basic idea is that the automatic gear shifter would up-shift (increase the gear ratio) if the rider is pedaling too fast ($\omega > \omega_{\text{opt}} + \delta$), and down-shift (decrease the gear ratio) if the rider is pedaling too slowly ($\omega < \omega_{\text{opt}} + \delta$), where $\delta$ is a parameter that defines a window of acceptable pedaling velocities. Note that $\delta$ must be chosen appropriately given the ratio difference between adjacent gears, so that when the bike shifts it doesn’t “overshoot” the optimal velocity range and cause another shift in the opposite direction.

The optimal pedal velocity will depend on the effort level of the rider. Assume the rider selects a desired level of effort by tipping a tilt sensor on his head forward. The measured angle of the head could be used to specify the average pulse width duration of the stimulating current, roughly analogous to “effort”. In a set of preliminary measurements, you could measure the maximum angular velocity achieved for each of the levels of effort (i.e. pulse width durations), when pedaling against a small load (for example, when pedaling with the rear wheel of the bicycle set on low-friction rollers). Then, when the rider selects a level of effort $e$, you would know the desired angular velocity of the legs. Specifically, $\omega_{\text{opt}} = 0.3*\omega_{\text{max}}(e)$, where $\omega_{\text{max}}(e)$ is the previously measured maximum angular velocity at zero load for effort level $e$.

An electronic gear shifter could be designed that would take as inputs the current level of effort and current pedal crank velocity, then specify whether to down-shift or up-shift the gears:

\[
\begin{align*}
\text{AUTOMATIC SHIFT CONTROLLER} \\
&\text{Look-up table from initial effort/}\omega_{\text{max}}\text{ experiment} \\
&\begin{array}{l}
\omega > 0.3 \omega_{\text{max}}(e) + \delta \\
\Rightarrow \text{upshift}
\end{array} \\
&\begin{array}{l}
\omega < 0.3 \omega_{\text{max}}(e) + \delta \\
\Rightarrow \text{downshift}
\end{array}
\end{align*}
\]

4. Seat Height
As described in class and in the McMahon chapter, there exists an optimal length at which isolated muscle develops maximum force. At this length the thin and thick filaments of muscle are optimally overlapped. Pushing on a bicycle pedal involves using many muscles, each with its own optimal length and its own (changing) moment arm. However, it would be expected that the combination of synergist muscles working together to push the pedal downward would also exhibit maximal force generation at some optimal leg configuration (i.e. some hip and knee angle). It makes sense to adjust the seat height so that the leg is in this optimal configuration midway through the pedaling down stroke.
To find the appropriate seat height, a stationary bicycle could be instrumented to measure pedal velocity and pedal force at varying seat heights and pedal velocities. The optimal seat height could be estimated by locking the pedal (i.e. setting its velocity to zero) at horizontal (midway through the down stroke), adjusting the seat to several heights (for example, low, high, and medium), and measuring maximal force generation during electrical stimulation of the customer's leg muscles (equivalent to an isometric force-length experiment). By fitting a curve to the three data points, and finding the maximum of the fitted curve, the optimal seat height could be estimated. Note that the optimal seat height must also meet the condition that the leg can turn the crank through its full range.

- Problem: Find the value of \( v \) that maximizes the power output of muscle described by the Hill equation. Assume \( \frac{\gamma_L}{\gamma_T} = 0.25 \) and show that maximum power is about 0.15 \( \gamma_T \).

\[
P = \frac{\gamma_L (v_0^* - av)}{v_0^* + b} \quad \text{(from the Hill equation, \( P = Fv \))}
\]

Find \( v \) which maximizes \( P \). At max \( P \),

\[
\frac{dP}{dv} = 0 = \left( \frac{v_0^*}{v_0^* + b} \right) \left( \frac{1}{v_0^* + b} \right) v_0^* (v_0^* - av) - \frac{\gamma_L (v_0^* - av)}{v_0^* + b}^2
\]

\[
\Rightarrow \begin{align*}
2a v_0^* - 2a v^2 - 2a b v + b^2 T_0 &= 0 \\
2a v_0^* - 2a b v + b^2 T_0 &= 0
\end{align*}
\]

\[
\frac{v_0^*}{v_0^* + b} \frac{\gamma_L}{v_0^* + b} = \frac{b}{a}
\]

\[
\Rightarrow b = 0.25 \gamma_T
\]

\[
v = \frac{1}{2} \gamma_T \quad \text{at} \quad \gamma_T = 0.5 \gamma_T
\]

\[
V_{opt} = \frac{3}{2} \gamma_T \gamma_{max} = 0.31 \gamma_T \quad \text{Velocity of optimal power}
\]

Can solve for power at \( V_{opt} \) using (1)

\[
P = \frac{V_{opt} (v_0^* - a \gamma_{opt} T_0)}{v_0^* + b}
\]

\[
= 0.31 \gamma_T \left( 0.25 \gamma_{max} T_0 \right) = 0.25 \gamma_T \left( 0.31 \gamma_{max} \right)
\]

\[
P = 0.08 \gamma_{max} T_0 \quad \text{at} \quad V_{opt}
\]