EECS 114: Assignment 3

October 20, 2010

Due Tuesday 2 Nov 2010 at 11:00am

This assignment is designed to help prepare you for the mid-term exam.

1. Express the function \( n^3 / 1000 - 100 n^2 + 100 n + 3 \) in terms of \( \Theta \)-notation.

2. Rank the following functions by order of growth: \( \log n, n!, n^3, n \log n, 2^n, n \).

3. What is the time complexity of following code segment? Support your answer by analyzing the steps taken in each round.
   
   ```java
   for (i = 0; i < n; i++)
       for (j = 0; j <= i; j++)
           K++;
   ```

4. Show that for any real constant \( a \) and \( b \), where \( b > 0 \), \( (n + a)^b = \Theta(n^b) \).
5. Use mathematical induction to show that when \( n \) is an exact power of 2, the solution of recurrence
\[
T(n) = \begin{cases} 
2 & \text{if } n = 2, \\
2T(n/2) + n & \text{if } n = 2^k, \text{ for } k > 1
\end{cases}
\]
is \( T(n) = n\log n \).

6. What would be the resulting sequences of the pre-order, in-order and post-order traversals of the following tree?

```
          A
         / \
        B   C
       /   / \
      D   E   F
     /     /   / \
    G     H   I
```

pre-order: ____________________________
in-order: ____________________________
post-order: ____________________________

7. Construct a binary search tree from the values shown below. The tree must be drawn in the order the values appear.
Input values: 40, 35, 60, 74, 58, 37, 70, 36, 20, 50.

8. How can we use a binary search tree to sort a list of numbers in an ascending order?
9. Give a pseudo code for finding the height of a binary tree implemented as a pointer-based data structure.

10. Assume we have a hash function: \( h(x) = x \mod 13 \) and a hash table of size 13. Linear probing is used to handle collisions.
   A) Show the hash table after the following numbers are entered: 3, 12, 4, 17, 11, 16, 10
   ![Hash Table](image)

   B) Continue A by deleting 17.
   ![Hash Table](image)

   C) Continue B by inserting 18.
   ![Hash Table](image)

11. Given the skip list below:

   ![Skip List](image)

   A) Show the process of searching for 50 in this skip list.
B) Draw the skip list after adding 65 to the list such that a new level is added to the list.

12. Provide a pseudo code for finding the largest value in an array using the divide-and-conquer technique.
   *Hint:* The largest element in an array is the maximum of the largest element of its sub-arrays.

13. Sort the following sequence using Merge Sort. Illustrate your results after each pass.
   \(< 25, 6, 15, 31, 3, 11, 18, 9, 19 >\)
14. What is the worst-case time complexity of Quicksort. How could it happen? Provide a work-around for this problem.

15. Give a brief argument that the running time of PARTITION (used in Quicksort) on a subarray of size n is Θ(n). The PARTITION procedure is given in Section 7.1 of the textbook.

16. Illustrate the operation of Radix-sort on the following list of English words: COW, DOG, SEA, RUG, ROW, BOX, TAB, BAR

17. Would you implement a priority queue using the bottom-up construction approach? Why or why not?
18. Consider the following min-heap:
A) Draw the resulting tree after inserting 0 into the min-heap.

B) Draw the resulting tree after removing 1 from the min-heap (independent of A).

19. A) What are the minimum and maximum numbers of elements in a heap of height \( h \).

B) What are the minimum and maximum numbers of leaves in a heap of height \( h \).

20. Provide an example showing that the greedy algorithm of the fractional knapsack problem does not find the optimal solution for the non-fractional (0/1) knapsack problem.