(a) $X \sim \text{Geometric} (p)$

\[ p = \Pr (X = 11) = \frac{8}{36} = \frac{2}{9} \]

(b) $Y \sim \text{Neg Bin} (100, p)$

\[ \mathbb{E}X = \frac{1}{p} = 4.5 \]

\[ \mathbb{E}Y = \frac{100}{p} = 450 \]

(c) $Y = \sum_{i=1}^{100} X_i$ where $X_i \sim \text{Geometric} (p)$

\[ \text{Var}(Y) = \sum_{i=1}^{100} \text{Var}(X_i) = 1400 \]

(d) $Y \sim N (450, 1400)$ by CLT

\[ \Pr (Y \leq 500) = \Pr \left( \frac{Y - 450}{\sqrt{1400}} \leq \frac{50}{\sqrt{1400}} \right) \]

\[ = \Pr (Z \leq 5) \]

(e) Let $V = \sum_{i=1}^{10} Y_i$ where $Y_i \sim \text{Binom} (10, \frac{2}{3})$

\[ \Pr (V = k) = \sum_{k=0}^{10} \binom{10}{k} \left( \frac{2}{3} \right)^k \left( \frac{1}{3} \right)^{10-k} \]

$\lambda \neq 0$

\[ \mathbb{E}X^2 = \int X^2 f_X (X) \, dX \]

\[ \mathbb{E}(X-1)^2 = \sum_{x=0}^{\infty} (x-1)^2 \frac{\lambda^x e^{-\lambda}}{x!} \]

\[ = \lambda^2 \frac{e^{-\lambda}}{2} \]

\[ \Rightarrow \mathbb{E}X^2 - \mathbb{E}X = \lambda^2 = \mathbb{E}X^2 - \lambda \]

\[ \Rightarrow \mathbb{E}X^2 = \lambda^2 + \lambda \]

\[ \Rightarrow \text{Var} X = \lambda^2 + \lambda - \lambda^2 = \lambda \]
P. 6

(a) \[ P(\text{all } R) = \frac{\binom{3}{1}}{\binom{10}{2}} \]

(b) \( P(\text{all } C \text{ or all } R) = ? \)

But: \( \{\text{all } C\} = \{\text{all } R\} \)

\( \implies \{\text{all } C\} \text{ or } \{\text{all } R\} = \{\text{all } R\} \)

So, same as (a).

(c) \( P(X = 1) = 1 - P(X = \text{NDR}) = 1 - \frac{\binom{4}{1}}{\binom{10}{2}} \)

(d) \( P(\text{ex } 2 \text{ C and ex 1D}) = \frac{\binom{4}{2}\binom{1}{1}}{\binom{10}{2}} \)

(e) \( P(\text{chain } \text{ in } C \text{, } \text{VC } \text{ is } L \text{, } \text{else cannot}) \)

\[ = \frac{4 \cdot 3 \cdot \binom{3}{2}}{10 \cdot 9 \cdot \binom{8}{2}} \]

P. 10

(a) Let \( X = \) # shrubs that die out of 25

\( \text{Bin}(25, 2) \)

\[ P(X = 0) = \binom{25}{0} \cdot 0.5^{25} \approx 0.00000025 \]

(b) \( P(X = 15) = \binom{25}{15} \cdot 0.5^{25} \approx 0.00000025 \)

(c) \( P(X = 0) = 1 - P(X = 15) \)

(d) \( P(\text{ex } 3 \text{ die in 2nd year}) = \)

\[ \sum_{j=0}^{25} P(\text{ex } 3 \text{ die in year } j, \text{ ex } j \text{ die in year } j) \cdot \text{Pr}(X) \]

\[ = \sum_{j=0}^{23} P(\text{ex } 3 \text{ die in year } j, \text{ ex } j \text{ die in year } j) \cdot \text{Pr}(X) \]

\[ \text{ex } j \text{ die in year } j \]
Given that all 25 - j plants alive, that 25 - j that die in 42 is Bin

Dogs

# ways to place dogs in pens as pairs:

\[
\binom{8}{2} \binom{6}{2} \binom{4}{2} \binom{2}{2} = \binom{8}{2} \binom{2}{2} = \binom{2222}{8}
\]

Place 2 dogs in Pen 1, in Pen 2.

The only way to set ex 6 dogs with fleas is to have one pen w/ 2 dogs that already have fleas, one pen w/ 2 dogs that don't have fleas, and 2 remaining pens get one of each. Put 2 dogs w/ fleas in selected pen

# ways = \binom{4}{1} \binom{3}{1} \binom{8}{1} \binom{4}{2} \binom{1}{1} \binom{1}{1} \binom{1}{1} \binom{1}{1}

Pick a pen, pick a pen for 2 w/ fleas, for 2 w/o fleas.

Place 2 dogs w/o fleas in selected pen

Place other 4 dogs in remaining 2 pens so that 1 of each in both.