2.2. Consider a mobile radio environment in which we model only path loss and Rayleigh fading. The path loss exponent is $\eta$. The transmit power, averaged over Rayleigh fading, at the reference distance $d_0$ from a transmitter is $P$.

a. Write down an expression for the random received power $P_{\text{rcv}}(d)$ at a receiver at a distance $d = ad_0$, and obtain the distribution of $P_{\text{rcv}}(d)$.

b. Two cochannel transmitters (indexed 1 and 2) are simultaneously transmitting at distances $d_1 = a_1d_0$ and $d_2 = a_2d_0$ from the receiver. A transmission can be decoded if its signal to interference ratio exceeds $\gamma$. Ignoring the receiver noise, obtain the probability that the transmission from Transmitter 1 is decoded, treating the signal from Transmitter 2 as interference. This is called the capture probability (of Transmitter 1 over Transmitter 2).

c. Determine $\beta$ such that if $a_2 > (1 + \beta)a_1$ then the probability of transmission 1 being decoded is greater than $1 - \epsilon$ ($\epsilon > 0$ is very small).

Solution:

a. The received power at distance $d$ is modeled as $P_{\text{rcv}}(d) = P \left( \frac{d}{d_0} \right)^{-\eta} R^2$ where $R^2$ is exponentially distributed with mean 1. Hence $P_{\text{rcv}}(ad_0)$ is exponentially distributed with mean $Pa^{-\eta}$; i.e., $\Pr(P_{\text{rcv}}(ad_0) > x) = e^{-x/(Pa^{-\eta})}$.

b. The powers received from the two transmitters are $P_{1,\text{rcv}} \sim \exp(1/(Pa_1^{-\eta}))$ and $P_{2,\text{rcv}} \sim \exp(1/(Pa_2^{-\eta}))$ (where we use $\sim$ to denote “distributed
as"). The probability of decoding Transmitter 1 is
\[
P \left( \frac{P_{1,\text{rev}}}{P_{2,\text{rev}}} > \gamma \right) = P(P_{1,\text{rev}} > \gamma P_{2,\text{rev}})
\]

Now
\[
\gamma P_{2,\text{rev}} \sim \exp \left( \frac{1}{\gamma P_{2,\text{rev}}} \right).
\]

Hence the probability of decoding Transmitter 1 is
\[
\frac{1}{\gamma P_{2,\text{rev}}} + \frac{1}{P_{1,\text{rev}}} = \frac{1}{1 + \left( \frac{\omega}{a_2} \right)^{-\eta}}
\]

which, as expected, goes to 1 as \( \frac{\omega}{a_2} \to 0 \).

c.

\[
\frac{1}{1 + \gamma \left( \frac{a_2}{(\frac{a}{a_2})} \right)^\eta} \geq \frac{1}{1 - \gamma (1 - \beta)^{-\eta}} > 1 - \epsilon
\]

Hence \( \beta > \sqrt{\frac{\gamma(1-\epsilon)}{\epsilon}} - 1 \).

2.3. Consider the binary modulation scheme analyzed in Section 2.1.1. Obtain the bit error rates for various SNR values \( \gamma = 12 \text{ dB}, 11 \text{ dB}, 10 \text{ dB}, \) and 9 dB. In each case, calculate the probability of packet error for 1500 byte packets. Hence compare the plots in Figure 3.9 with the AWGN plot in Figure 3.12. Hint: Use the approximation \( Q(x) \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \).

Solution: With the white Gaussian noise assumption, the bit errors in a packet are independent. For a bit error probability of \( \epsilon \), the packet error probability for a packet of length \( n \) bits is \( p = 1 - (1 - \epsilon)^n \). The following table shows the calculated values.

For each value of \( \gamma \), read off the throughput for the corresponding packet loss probability from the TCP Tahoe plot in Figure 3.9, and see that it compares well with the value for that \( \gamma \) read off the AWGN plot in Figure 3.12.
<table>
<thead>
<tr>
<th>$\gamma \text{ dB}$</th>
<th>$\gamma$</th>
<th>$\epsilon$</th>
<th>$p$</th>
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<tr>
<td>12</td>
<td>15.84</td>
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<tr>
<td>11</td>
<td>12.59</td>
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<td>9</td>
<td>7.94</td>
<td>$3.55 \times 10^{-5}$</td>
<td>0.3472</td>
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</tbody>
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