LUMPED-CAPACITANCE HEAT TRANSFER

1 Introduction

The purpose of this experiment is to determine the heat transfer coefficient, \( h \), from a sphere under the ‘lumped capacitance’ or low Biot number heat-transfer assumption. A metal sphere is heated to a uniform temperature in boiling water, and at time \( t = 0 \) it is exposed to room air. A thermocouple is installed in the sphere, and the sphere temperature is recorded versus time. Under the low Biot number assumption the thermal conductivity of the sphere metal is extremely high, and the temperature decay is uniform throughout the sphere. The time-rate of temperature decay is controlled by the heat transfer coefficient from the surface of the sphere to the environment. As will be shown, the heat transfer coefficient can be determined from the temperature-time data.

The heat transfer between a solid and fluid can be separated into two main regimes: natural and forced convection. As the words imply, “natural” means that no external means are used to cause flow in the fluid. Any fluid motions are the result of buoyancy-induced effects in the fluid which arise from temperature differences between the object and the fluid in the earth’s gravitational field. “Forced” convection implies that the fluid is moved by an external means such as a blower or pump. Heat transfer to boiling liquids is treated separately.

The central problem is to describe the cooling (or heating) of a solid sphere initially at a uniform temperature subjected to natural or forced convection heat transfer at the surface. An immediate simplification can be made if the material of the sphere has a high thermal conductivity, such that the sphere cools or warms without internal temperature gradients. Then the temperature of the sphere is assumed to be spatially uniform, but changing with time. This approximation is known as the ‘lumped capacitance’ model (“Fundamentals of Heat and Mass Transfer,” Incopera and Dewitt (1990), pp. 226 ff.). The first-law balance between the rate of change of internal energy of the mass and the surface heat flux times the area of the sphere is:

\[
-mC_v \frac{dT_v}{dt} = q'' A_s,
\]

where \( m \) = mass of sphere, \( C_v \) = specific heat at constant volume (equal to that at constant pressure for a solid), \( T_v \) = bulk temperature of the sphere, \( t \) = time, \( A_s \) = surface area of the sphere, and \( q'' \) = surface heat flux. The mass of the sphere is of course equal to the density times the volume.

For forced convection, the surface heat flux is specified by Newton’s “Law” of Cooling,

\[
q'' = h(T_s - T_\infty),
\]

where \( h \) = heat transfer coefficient, \( T_s \) = surface temperature and \( T_\infty \) = environmental temperature of the fluid far away from the sphere. The heat transfer coefficient is actually an empirical parameter, not a universal constant (hence the quotes around “Law”). It is a function of the properties of the fluid, the shape of the object and the flow itself. (Be sure you understand the difference between \( h \) and \( k \), the thermal conductivity in Fourier’s law.) For natural convection, \( h \) varies as \((T_s - T_\infty)^n\), where \( n \) is an empirical number, but for low temperature differences Equation 2 is often applied,
i.e., $n = 1$. (The orientation of the object relative to the gravity vector is also important in natural convection, but of course not for the symmetrical shape of a sphere.) Typical values of $h$ for air are 3-9 W m$^{-2}$ K$^{-1}$ for natural convection and 30-100 W m$^{-2}$ K$^{-1}$ for forced convection.

For the lumped capacitance approximation, we can set $T_v = T_s$ in equation (1), and thus with equation (2) we obtain:

$$-mC_v dT_s/dt = hA_s(T_s - T_\infty).$$

Since $T_\infty$ is constant, and assuming that $C_v$ is independent of temperature, we can solve the first-order ordinary differential equation (3). We introduce a dimensionless temperature $\Theta$ defined as

$$\Theta = \frac{T_s - T_\infty}{T_i - T_\infty},$$

where $T_i$ is the initial temperature of the sphere. $\Theta$ will therefore vary between 1 and 0 for $T_s$ cooling from $T_i$ to $T_\infty$. Equation (3) becomes

$$\frac{d\Theta}{\Theta} = d(ln\Theta) = -\frac{hA_s}{mC_v} dt.$$

Since $h$ is assumed to be independent of $\Theta$, equation (5) integrates to

$$\Theta = C_1 e^{-\frac{hA_st}{mC_v}},$$

where $C_1$ is the constant of integration. At $t = 0$, $\Theta = 1$, and $C_1 = 1$. Therefore the solution to the cooling of sphere under the lumped capacitance assumption is

$$\Theta = e^{-\frac{hA_st}{mC_v}}.$$ (7)

The temperature everywhere inside of the cooling sphere is predicted to fall exponentially with time, and the heat transfer coefficient $h$ can be determined from the rate of temperature decrease since the quantities $A_s, m$ and $C_v$ are known for a given sphere. Equation 7 predicts that the time decay of dimensionless temperature is exponential. Exponential decay of temperature, voltage, etc., is common in many engineering problems. The usual form, in the context of the low Biot number problem, is:

$$\Theta = e^{-\frac{t}{\tau}},$$ (8)

where $\tau$ is the time constant, $\tau = \frac{mC_v}{hA_s}$. Good experimental practice is to allow the system to decay for at least one time constant, such that $\Theta = e^{-1} \approx \frac{1}{3}$.

Since $\frac{1}{3}$ is dimensionless, we can cast it in the form of products of dimensionless quantities. We define a characteristic length for an object as the ratio of the volume to the surface area:

$$L_c = \frac{V}{A_s},$$

where $V$ is the volume of the object. Noting that $m = \rho V$, we obtain

$$\frac{t}{\tau} = \frac{hA_s t}{\rho V C_v} = \frac{ht}{k \rho L_c C_v} = \frac{hL_c}{k \rho C_v L_c^2} = \frac{hL_c \alpha t}{k L_c^2},$$ (9)

where $\alpha$ is the thermal diffusivity of the material of the object, $\frac{k}{\rho C_v}$.
The dimensionless quantity $\frac{hL_c}{k}$ is known as the Biot number, and is a measure of the relative importance of convective heat transfer at the surface to conduction in the solid object:

$$Bi = \frac{hL_c}{k}.$$  \hspace{1cm} (10)

For the lumped capacitance approximation to be valid, the thermal conduction in the object must be very large or the heat transfer to the fluid small, expressed as $Bi << 1$.

A short biography of Biot is given in the following link:

http://www-groups.dcs.st-and.ac.uk/history/Mathematicians/Biot.html

2 Apparatus (see Fig. 1)

The purpose of the present experiment is to measure the rate of cooling of a metal sphere under natural and forced convection, determine if the temperature decay with time follows the form of Equation 7, and, if so, calculate $h$.

The apparatus consists of brass and aluminum spheres with type K thermocouples embedded in the center. The thermocouple is connected to a National Instruments’ data system. Another thermocouple monitors the ambient temperature. Chose a sample rate of 1 per second. Store the data to a file for import later into Excel. The data are simply the two temperatures and their difference versus time. We expect an exponential type change of the sphere temperature with time. How long should we take temperature-time data? For the theoretical limit of equilibrium of the sphere temperature with ambient environment, we would have to wait for an infinite time. No one wants to stay in lab that long! Since all we observe is the sphere temperature changing with time, we have to decide when to stop the data recording based on the sphere temperature alone. We could use an arbitrary rule, like 50% of the initial temperature or something equivalent, but the theory should enable us to make a more rational choice. You must make a good engineering ‘back-of-the-envelope’ calculation, based on the theory, of a reasonable final temperature for the end of a data run. Do this before the laboratory quiz and give it to the TA.

Property Values for Brass
Thermal Conductivity: 111 W/mK
Specific Heat Capacity $C_v$: 385 J/kgK
Density: 8520 kg/m$^3$
Thermal Diffusivity: $3.384 \times 10^{-5}$ m$^2$/sec
Sphere Radius: 0.0254 m (1 inch)

Property Values for Aluminum
Thermal Conductivity: 237 W/mK
Specific Heat Capacity $C_v$: 903 J/kgK
Density: 2702 kg/m$^3$
Thermal Diffusivity: $9.71 \times 10^{-5}$ m$^2$/sec
Sphere Radius: 0.0254 m (1 inch)
3 Procedure

Become familiar with the National Instruments’ data program. Set the sample rate to 1 per sec if it isn’t already. Use a logical path and file name for the data files to be written to, e.g., c:\MAE107\Lab_Day\Biot\Brass_boil.dat. Copy a test file from the lab computer to the floppy drive or USB port before starting the actual data collection to insure that the data can be retrieved. The data are written in ASCII format and are readily imported into Excel, Matlab, etc.

3.1 Boil Water

With water in the pot, place the free thermocouple in the water so it does not touch the wall of the pot, start the data program and then the hot plate. Watch the time-temperature plot until the water boils vigorously. Is the temperature behavior what you expected?

3.2 Boil Sphere

This run will allow you to determine the heat transfer coefficient between a sphere and boiling water.

Bring the water in the pot to a boil. Activate the National Instruments’ thermocouple software. Make the first run while heating either the brass or aluminum sphere from ambient temperature to boiling. Quickly plunge the sphere into the boiling water and record data until your estimate of the final temperature is reached. Then stop the data recording and copy the file to the appropriate directory on the hard disk.

Repeat for other sphere, e.g., if you started with the brass sphere now use the aluminum one. After the data recording is stopped, leave the sphere in the boiling water in preparation for the next experiment. Replenish the water in the pot as necessary to keep the sphere completely covered in boiling water.

3.3 Forced Convection to Air

(Each group MUST calculate the heat transfer coefficients for their data and show to the TA before leaving)

Move the ambient thermocouple from the pot and place it in the entrance section of the wind tunnel. Set the wind tunnel airspeed to \( \sim 20 \text{ m s}^{-1} \) and record the manometer reading. With a sphere (note whether brass or aluminum) at the temperature of the boiling water, start the data recording and quickly remove the sphere by the handle from the pot and place it in the center of the tunnel test section. Record data until at least the pre-estimated final temperature is reached.

Repeat for the other sphere.

3.4 Natural Convection to Air

Place the ambient thermocouple in the vicinity of the ring stand, but from the sphere location. With a sphere at the temperature of the boiling water, start the data recording and quickly remove the sphere by the handle from the pot and place it on the ring stand away from air currents. Record data until the pre-estimated final temperature is reached. (One group to do a brass sphere, the other aluminum; make sure you coordinate between groups to insure this.)

Obtain the data for the other sphere from the other lab group.
3.5 Summary of Required Runs

The required 6 data runs are:

<table>
<thead>
<tr>
<th>Run #</th>
<th>Sphere</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T/C</td>
<td>Boil</td>
</tr>
<tr>
<td>2</td>
<td>Br or Al</td>
<td>Heat in Boiling Water</td>
</tr>
<tr>
<td>3</td>
<td>Al or Br</td>
<td>Heat in Boiling Water</td>
</tr>
<tr>
<td>4</td>
<td>Br or Al</td>
<td>Cool Air Forced (Wind Tunnel)</td>
</tr>
<tr>
<td>5</td>
<td>Al or Br</td>
<td>Cool Air Forced (Wind Tunnel)</td>
</tr>
<tr>
<td>6</td>
<td>Br or Al</td>
<td>Cool Air Natural#</td>
</tr>
<tr>
<td>7</td>
<td>Al or Br</td>
<td>Cool Air Natural#</td>
</tr>
</tbody>
</table>

#One group to do Br, the other Al.

4 Analysis
The main point of the lab is to obtain h from the measured cooling rate, dT/dt, for the different conditions. Your group should develop the experimental and analysis procedures to do this in the laboratory period. Transfer the data from the National Instruments’ computer to one of the analysis computers via a floppy or internet and import the data into Excel.

From the tabulated data, calculate $\Theta$ and plot ln$\Theta$ versus time, t. Obtain h from the plot. (You may have to ignore the initial few seconds of data.) Show the results to your TA.

5 3-D Numerical Analysis using SolidWorks
Thermal Analysis is now available in SolidWorks Simulation. 3-D results can be obtained by numerically solving the heat transfer governing equation, which is a partial differential equation. Please refer to Heat Transfer Textbook for the equation and SolidWorks help for using the simulation.

6 Mandatory Questions
Write a Summary of the experiment. As always, include the major quantitative results. [10 points]
Sample calculations with unit conversions must be shown in the report or Appendix. (All units must be shown; h should be expressed in SI units of W m$^{-2}$ K$^{-1}$.)

1. Estimate the boiling temperature for Denver, CO? [5 points]

2. Turn in plots of $T_s$ versus t and ln$\Theta$ versus t for each run. Several runs on each plot, with legend, or multiple small plots on one page are OK. Clearly show the values of $\tau$ on the ln$\Theta$ versus t plots and find a way to use the curve of ln$\Theta$ versus t to calculate h [15 points]

3. Theoretically, you can use any point ($T_s$, t) to calculate h (Eq. 7). From the development of Eq. 7 and the exp. design, what do you think may contribute to the errors (not related to the exp. equipments) in calculating h? How to minimize the errors? [10 points]

4. Compute the Biot number for each run. Are the Biot numbers small enough to justify the lumped capacitance assumption? [5 points]
5. Do you expect the heat transfer coefficients $h$ to be different for the brass and aluminum spheres under the same conditions, e.g., forced convection, boiling, or natural convection? If yes, why? If no, why? Compare your results for the spheres of brass and aluminum and comment. [10 points]

6. Compare your values of $h$ to those in heat-transfer references. You may have to go the library. [5 points]

7. From the basic measurements, if using Eq. 7 and any point $(T_s, t)$ to calculate $h$, estimate $\delta h$ using $\delta T$ and $\delta t$. Please comment on how to minimize $\delta h$. [15 points]

8. Please use Solidworks to reconstruct a sphere and run the simulation and plot the temperature profile change with time $(T(r,t))$. Please use one data set of cooling to compare with the analytical result (Eq. 8) and simulation result (using the center temperature). For simulation, please reduce the thermal conductivity by 100, please replot $T(r,t)$ and comment on the result. [15 points]

9. The TES tank at the CP is a large cylinder 105 feet high and 88 feet in diameter to store the excess chilled water. Estimate the heat loss from the tank for winds of 0 and 20 ms$^{-1}$. Take the surface the tank to be 40°F and the air temperature to be 120°F. For a 12-hour period, compare energy loss to “that” stored in the tank. Would more insulation be worth it? Use the heat transfer coefficients from your experiments. [10 points]

10. Use ~50 data points around the time constant in your measurement of the forced convection for the BRASS sphere cooling to calculate $h$ (~50). Plot the frequency distribution of $h$ (please choose a proper bin size). Make a comment on the Histogram. [5 points]

Fig. 1 The lab setup.