1 Introduction

A wind tunnel provides a controlled air stream to study aerodynamic and fluid flow phenomena. Most wind tunnels are designed to have a uniform velocity profile in the test section with a low turbulence level. The test object is placed in the test section, and flow visualization or quantitative measurements are obtained.

The most common device for measurement of test-section flow speed is the Pitot tube, named after Henri Pitot who invented the device and used it to determine the flow rate of the Seine River in Paris in 1732. The Pitot tube is a small diameter tube with the open end facing the flow. Bernoulli’s equation is applied to a flow streamline that stagnates at the hole in the tip of the tube.

Bernoulli’s equation for steady, incompressible, in-viscid flow along a streamline is:

\[ P + \frac{1}{2} \rho V^2 + \gamma z = \text{const}, \]  

where \( P \) is the pressure, \( \rho \) is density, \( V \) is velocity, \( \gamma \) is specific weight, and \( z \) is elevation. Application of Bernoulli’s equation to a horizontal streamline which stagnates at the center hole of the Pitot tube yields:

\[ P_1 + \frac{1}{2} \rho V_1^2 = P_2 + 0, \]  

where \( P_1 \) and \( P_2 \) are the pressures where the velocity is \( V_1 \), and at the center hole of the Pitot tube where \( V_2 = 0 \), respectively. The total pressure at the center hole of Pitot tube is called the stagnation pressure, the sum of the static (\( P_1 \)) and dynamic (\( \frac{1}{2} \rho V_1^2 \)) pressures.

For a Pitot tube Bernoulli’s equation is rearranged as

\[ V = \sqrt{\frac{2\Delta P}{\rho}}, \]  

where \( \Delta P \) is \( P_2 - P_1 \), the difference between the Pitot stagnation pressure and the static pressure in the free-stream. The Pitot tube is therefore a simple device for the measurement of velocity under conditions for which the Bernoulli equation is applicable.

Before you come to the laboratory, develop a working equation for simple in-lab calculations of velocity from the measured pressure difference in inches of \( H_2O \) from the manometer. This will be of the form:
\[ V = K \sqrt{\Delta H}, \]  

(4)

where the manometer difference, \( \Delta H \), is in inches of water and \( K \) is a constant you calculate which is for dry air at standard conditions and expresses \( V \) in meters per second, \( ms^{-1} \). Turn this into the TA before the quiz. You can take the barometric pressure and temperature of air to be those of standard conditions.

The calculation of velocity from Bernoulli’s equation requires the measurement of the pressure difference \( \Delta P \). A large pressure difference is typically measured more accurately than a small one. In a measurement we are often interested in the sensitivity of the instrument. The sensitivity is the amount of change in the variable we measure per change in the input variable to the instrument. For the Pitot tube, the sensitivity can be defined as the change in measured pressure per change in velocity. This is obtained from Bernoulli’s equation as:

\[ S = \frac{d\Delta P}{dV} = \rho V. \]

(5)

Therefore, \( S \) increases with velocity; i.e., the sensitivity is not constant with \( V \), which of course is due to the non-linear Bernoulli equation. The above equation shows that the Pitot tube is therefore best suited to the measurement of reasonably high flow speeds as on aircraft.

The stagnation pressure is easily measured with a manometer or pressure transducer connected to the center hole of the Pitot tube. The static pressure could be measured on the streamline forward of the Pitot with a small tube with side holes in it. However, the insertion of a small tube upstream of the Pitot tube will disturb the streamlines. Therefore, the Pitot-static tube was developed. This variation of the Pitot tube has a concentric tube with static pressure holes in the outer tube at 12 to 24 tube diameters back of the tip. At these locations, the static pressure has “recovered” to the free-stream value from the distortion at the tip. The inner tube is the Pitot tube, and both pressures are readily connected to a differential manometer or transducer to obtain \( \Delta P \).

A Pitot-static tube is more difficult to make than a simple Pitot tube. For wind tunnel applications, we recall that for uniform flow the Bernoulli constant is the same across all streamlines. If we ignore the small pressure drop across the high Reynolds number boundary layer on the walls of the straight wind tunnel test section, then we can simply put a small hole in the test section wall to serve as the static pressure tap. The same arrangement is often used on aircraft, where the Pitot tube and static pressure taps are often located at different places. The static pressure taps are located on the fuselage at locations where the measured pressure is close to the free-stream pressure. Static ports on both sides of the fuselage are manifold-ed together so that a correct pressure is obtained when the aircraft yaws. The static pressure is also used to calculate the altitude referenced to a standard atmosphere. The Pitot tubes stick out into the air-stream and are heated in case of icing.
conditions. The next time you fly on an airliner, look at the Pitot tubes at the front of the plane when the plane is parked at the gate. There are several for redundancy. Depending on the aircraft model, you may also be able to see the flush static pressure ports, which are usually in an unpainted circle on the fuselage with a warning label to keep the ports clean. Note that there have been several fatal aircraft accidents in recent years due to plugged Pitot tubes and static ports. The aircraft autopilots and/or the real pilots could not cope with the bizarre airspeed/altitude data from plugged ports and sent the aircraft into uncontrollable dives and stalls.

2 Apparatus

Our apparatus consists of a small open-return wind tunnel. Room air is drawn into the test section through a smooth contraction section. The blower is located downstream of a honeycomb flow straightener to reduce swirl. Test section speed is varied by throttling the outlet of the constant-speed blower.

A simple Pitot tube on a traverse is connected to a liquid manometer, which contains a special low volatility oil and has a scale calibrated in inches of water. (An electronic transducer may also be used.) Air density will be required for your calculations. A barometer and thermometer are in the lab. Assume dry air.

Drawings of a Pitot tube, static tube, and a Pitot-static tube are shown in Figure 1 at the end of the handout. The principle of the manometer is shown in Figure 2. Note that the laboratory manometers are (1) tilted for better resolution, and (2) are filled with an oil. Fortunately, the manufacturer has taken both effects into account, and the scale reads directly in inches of water so that you need not be concerned with these factors (provided the manometer is leveled as indicated by the bubble).

3 Procedure

3.1 Bernoulli’s Equation

In this portion of the experiment, two different applications of Bernoulli’s equation will be used to measure the flow speed. Position the Pitot-static tube in the center of the tunnel.

1. Measure the pressure difference between the Pitot pressure from the Pitot-static tube and the wall static tap. (Do not hook up the static tube of the Pitot-static to the manometer.) (i.e. the Pitot-wall tap, see Fig.3)

2. Leave the tunnel speed fixed, and carefully remove the plastic tubing line from the Pitot tube. Observe and record the pressure difference between the open tube and the wall static tap.
Think about the results from (1) and (2). If the pressures are the same, within experimental uncertainty, why? Turn off the tunnel and discuss with your TA before proceeding.

### 3.2 Velocity Profile

Use the Pitot tube and static wall tap arrangement to traverse the test section and obtain profiles of the velocity across the section. You may use a non-uniform spacing of the profile data points with higher resolution in the boundary layers. Enter the data into Excel and plot the velocity profile. Compute the average velocity and indicate it on the plot.

### 3.3 Cylinder Wake

Place a plastic cylinder in a center hole upstream of the Pitot tube. Mount the Pitot tube on the machinist’s height gauge and note the scale readings and units. Set a reasonable tunnel air speed (manometer at least 1.0 in water).

Make a preliminary traverse of the cylinder wake to insure that the minimum Pitot pressure is measurably above 0. If the Pitot is too close the cylinder, a region of reverse flow may be present. A Pitot tube does not work in such a situation. If the minimum Pitot pressure is too low, move the cylinder to another hole farther upstream.

Carefully profile the wake of the cylinder recording the position of the Pitot tube from the height gauge and manometer $\Delta H$. Obtain closely-spaced readings where the velocity gradients are largest. This will be important when you perform the required analyses.

Repeat with the cylinder at a different distance from the Pitot tube.

### 4 Questions

Always show sample calculations with units in an Appendix or in the body of the report.

#### 4.1 Mandatory

1. **Summary.** Write a Summary of the experiment. [15 points]

2. **Bernoulli’s Equation.** Are the pressures obtained from the Pitot-wall tap and open tube-wall tap measurements the same? Explain your answer in terms of Bernoulli’s equation. A sketch may help. [15 points] (See Fig. 3)

3. **Boundary Layer.** Why and under what conditions may we confidently neglect the pressure drop across the thickness of the boundary layer on the wind tunnel wall when using the wall static pressure tap? [15 points]
4. **Velocity Profile.** Plot the velocity profile in dimensional form, and also in non-dimensional form with the velocity normalized with the center-line, maximum value and the test section position by the section width. Integrate the profile to obtain the flow rate and average velocity and plot the average velocity as a separate line on the appropriate plot. [15 points]

5. **Wake and Drag.** Plot the velocity distribution in the cylinder wake with the velocity normalized with the free-stream value and the distance by the diameter of the cylinder. Plot the normalized momentum deficit versus non-dimensional distance. Obtain the drag on the cylinder from the momentum deficit profile by numerical integration. Calculate the drag coefficient and compare to results in your textbook or other reference. [40 points] (Note "distance by the diameter" means we use the diameter of the cylinder to normalize the distance, i.e. y/D)

### 4.2 For your own interest

1. **Exact Manometer Equation.** The usual manometer formula relates the difference in heights of the manometer fluid $\Delta H$ to the pressure via $\Delta P = \rho_m g \Delta H$, where $\rho_m$ is the density of the manometer fluid. The fact that there is another fluid, in our case air, on the depressed side also of height $\Delta H$ does not appear. Work out the complete manometer equation and estimate the error as a function of $\Delta H$ for an air-water manometer. Make a plot of the error versus $\Delta H$ for $\Delta H \leq 1$ inch water.

2. **Sensitivity and Error Analysis.** In the first experiment in MAE 107, you developed the estimated error for velocity from Bernoulli’s equation for an incompressible fluid.

   “Derive the predicted root-mean-square error for the velocity, $\delta V$, as determined from the application of Bernoulli’s equation for a constant density fluid, in terms of the error in the measured pressure difference. Denote the error in $\Delta P$ as $\delta P$.” Re-examine that result and consult with your TA if you did not do it correctly.

   Extend the analysis to consider errors in air density due to finite precision of the atmospheric pressure and temperature required to obtain the density, treating air as a dry ideal gas.

   You can assume that air is a dry ideal gas at the measured barometric pressure $P_s$ and temperature $T$. You will have to estimate the precisions of $\Delta P_s$ and $\Delta T$ from the instrument’s scales.

   Start with the following analysis:

   $$ V = V(\Delta H, \rho) $$

   $$ \delta V = \left( \frac{\partial V}{\partial \Delta H} \right)_\rho \delta(\Delta H) + \left( \frac{\partial V}{\partial \rho} \right)_\Delta H \delta \rho, $$

   (7)
where $\Delta H$ and $\rho$ are the independent variables of the function $V$, and $\delta(\Delta H)$ and $\delta\rho$ denote the estimated errors in $\Delta H$ and $\rho$ which result in the error in $V$, $\delta V$. Note that the function relating $V$ to the measured $\Delta H$ is non-linear and you should find that the error in $V, \delta V$, depends on the magnitudes of $\Delta H$ and $\rho$.

In principle, we should apply the same method to estimate the error in density, $\delta\rho$, from the dry air ideal gas law, namely,

\begin{align*}
\rho &= \rho(P_s, T) \\
\delta\rho &= \left( \left. \frac{\partial \rho}{\partial P_s} \right|_T \right) \delta P_s + \left( \left. \frac{\partial \rho}{\partial T} \right|_{P_s} \right) \delta T,
\end{align*}

where $\delta P_s$ and $\delta T$ are the errors in absolute barometric pressure, $P_s$, and temperature, $T$, respectively. These can be estimated as the precision of the readings from the barometer and thermometer in the laboratory.

The end result of your error estimate will be of the form

$$\delta V = A(\Delta H) \cdot \delta(\Delta H) + B,$$

where $A$ is a function of $\Delta H$. For the density of air at standard conditions, plot $\delta V$ as a function of $\Delta P$. Apply the estimated errors in $V$ to your results.
Figure 1: Pitot, Static and Pitot-Static Tubes.
Figure 2: Vertical Manometer.

Gage Pressure $\Delta P = P - P_0 = \rho gh$
Pitot tube
Static tube
Wall static tap
Pitot-wall tap: pitot tube+wall static tap
Open tube-wall tap: no connection of one hose to any tap+wall static tap

Figure 3