Chapter 7

Work and Energy
7-1 Work Done by a Constant Force

The work done by a constant force is defined as the distance moved multiplied by the component of the force in the direction of displacement:

\[ W = F d \cos \theta. \]

In the SI system, the units of work are joules:

\[ 1 \text{ J} = 1 \text{ N} \cdot \text{m}. \]

Work is a scalar quantity
Work can be positive, negative or zero
Examples of zero work

As long as this person does not lift or lower the bag of groceries, he is doing no work on it. The force he exerts has no component in the direction of motion.

The Moon revolves around the Earth in a nearly circular orbit, with approximately constant tangential speed, kept there by the gravitational force exerted by the Earth. The gravity does not do work.
A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_p = 100 \, \text{N}$, which acts at a $37^\circ$ angle as shown. The floor is smooth and exerts no friction force. Determine (a) the work done by each force acting on the crate, and (b) the net work done on the crate.
Definition of the scalar, or dot, product:

\[ \mathbf{A} \cdot \mathbf{B} = AB \cos \theta \]

Therefore, we can write:

\[ W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta. \]
Dot Products of Unit Vectors

• \( \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \)
  \( \hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0 \)

• Using component form with \( \mathbf{A} \) and \( \mathbf{B} \):

\[
\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\
\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \\
\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z
\]
A force \( \mathbf{F} = (6\mathbf{i} - 2\mathbf{j}) \text{N} \) acts on a particle that undergoes a displacement \( \Delta \mathbf{r} = (3\mathbf{i} + \mathbf{j}) \text{m} \). Find

- (a) the work done by the force on the particle and
- (b) the angle between \( \mathbf{F} \) and \( \Delta \mathbf{r} \).
Force done by constant forces: example
---- gravitational force

\[ \Delta W = m \vec{g} \cdot \Delta \vec{r} = -mg \Delta y \]

- \[ W = -mg(y_f - y_i) \]

independent of path.

\[ \Delta \vec{r} = \Delta x \vec{i} + \Delta y \vec{j} \]

\[ \vec{F} = -mg \vec{j} \]
Batman, whose mass is 80.0 kg, is dangling on the free end of a 12.0-m rope, the other end of which is fixed to a tree limb above. He is able to get the rope in motion as only Batman knows how, eventually getting it to swing enough that he can reach a ledge when the rope makes a 60.0° angle with the vertical. How much work was done by the gravitational force on Batman in this maneuver?
Particle acted on by a varying force. Clearly, $\vec{F} \cdot d$ is not constant!
For a force that varies, the work can be approximated by dividing the distance up into small pieces, finding the work done during each, and adding them up.

\[ W \approx \sum_{i=1}^{7} F_i \cos \theta_i \Delta l_i \]
In the limit that the pieces become infinitesimally narrow, the work is the area under the curve:

\[ W = \lim\limits_{\Delta \ell_i \to 0} \sum F_i \cos \theta_i \Delta \ell_i = \int_a^b F \cos \theta \, d\ell. \]

Or:

\[ W = \int_a^b \vec{F} \cdot d\vec{\ell}. \]
Example

- The force acting on a particle varies as in Figure. Find the work done by the force on the particle as it moves
  (a) from $x = 0$ to $x = 8.00$ m,
  (b) from $x = 8.00$ m to $x = 10.0$ m
  (c) from $x = 0$ to $x = 10.0$ m.
Example: work done by a spring force

The force exerted by a spring is given by: \( F_S = -kx \)

This is called Hooke’s Law

\[ W = \frac{1}{2} kx^2 \]
(a) A person pulls on a spring, stretching it 3.0 cm, which requires a maximum force of 75 N. How much work does the person do? (b) If, instead, the person compresses the spring 3.0 cm, how much work does the person do?
If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.
7-4 Kinetic Energy and the Work-Energy Principle

Energy was traditionally defined as the ability to do work. We now know that not all forces are able to do work; however, we are dealing in these chapters with mechanical energy, which does follow this definition.

We define the kinetic energy as:

\[ K = \frac{1}{2} mv^2. \]
If we write the acceleration in terms of the velocity and the distance: $v_2^2 - v_1^2 = 2ad$

The equation can be transformed to

$$v_2^2 - v_1^2 = 2F_{net}d / m \quad \text{or} \quad F_{net}d = \frac{1}{2}m(v_2^2 - v_1^2)$$

and finally to

$$W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2.$$
Work-Energy Principle

Work done is equal to the change in the kinetic energy:

\[ W_{\text{net}} = \Delta K = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2. \]

If the net work is positive, the kinetic energy increases.

If the net work is negative, the kinetic energy decreases.

Because work and kinetic energy can be equated, they must have the same units: kinetic energy is measured in joules. Energy can be considered as the ability to do work:
How much net work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s?
A horizontal spring has spring constant \( k = 500 \text{ N/m} \). (a) How much work is required to compress it from its uncompressed length \((x = 0)\) to \( x = 10.0 \text{ cm} \)? (b) If a 2.0-kg block is placed against the spring and the spring is released, what will be the speed of the block when it separates from the spring at \( x = 0 \), suppose \( F_D \) is 10N? (c) What is the largest speed of the block?
• An airplane pilot fell 400 m after jumping from an aircraft without his parachute opening. He landed in a snowbank, creating a crater 2.0 m deep, but survived with only minor injuries. Assuming the pilot's mass was 80 m/s and his terminal velocity was 50 m/s. Calculate
• The work done by the snow in bringing him to rest.
• The average force exerted on him by the snow to stop him.
• The work done on him by air resistance as he fell.
What should be the spring constant $k$ of a spring designed to bring a 1000 kg car to rest from a speed of 20 m/s so that the occupants undergo a maximum acceleration of 5.0 g?