Chapter 8

Conservation of Energy
8-1 Conservative and Nonconservative Forces

A force is conservative if:
the work done by the force on an object moving from one point to another depends only on the initial and final positions of the object, and is independent of the particular path taken.

Example: gravity.
Another definition of a conservative force:

*a force is conservative if the net work done by the force on an object moving around any closed path is zero.*
Nonconservative Forces

If friction is present, the work done depends not only on the starting and ending points, but also on the path taken. Friction is called a nonconservative force.
8-2 Potential Energy

An object can have potential energy by virtue of its surroundings.

Familiar examples of potential energy:

• A wound-up spring
• A stretched elastic band
• An object at some height above the ground

• Potential energy can only be defined for conservative forces.
Gravitational Potential Energy

In raising a mass $m$ to a height $h$, the work done by the external force is

$$W_{\text{ext}} = \vec{F}_{\text{ext}} \cdot \vec{d} = mgh \cos 0^\circ$$

$$= mgh = mg(y_2 - y_1).$$

We therefore define the gravitational potential energy at a height $y$ above some reference point:

$$U_{\text{grav}} = mgy.$$
If $U_{\text{grav}} = mgy$, where do we measure $y$ from?

It turns out not to matter, as long as we are consistent about where we choose $y = 0$. Only changes in potential energy can be measured.
A spring has potential energy, called elastic potential energy, when it is compressed. The force required to compress or stretch a spring is:

$$F_S = -kx,$$

where $k$ is called the spring constant, and needs to be measured for each spring.

$$U_{el}(x) = \frac{1}{2}kx^2.$$
Mechanical Energy and Its Conservation

If there are no nonconservative forces, the sum of the changes in the kinetic energy and in the potential energy is zero. This allows us to define the total mechanical energy:

\[ E = K + U. \]

And its conservation:

\[ K_2 + U_2 = K_1 + U_1. \]

The principle of conservation of mechanical energy
Problem Solving Strategy (no non-conservative forces)

— Conservation of Mechanical Energy

1. Define the isolated system and the initial and final configuration of the system
2. Identify the configuration for zero potential energy
3. Write the total energy as
   \[ E_i = K_i + U_i \] for the initial configuration
   \[ E_f = K_f + U_f \] for the final configuration
4. \[ E_i = E_f \] and you can solve for the unknown quantity
A bead slides without friction around a loop-the-loop. The bead is released from a height $h = 3.50R$.

(a) What is its speed at point A?
(b) What is the minimum value of $h$ for the bead to complete the circular motion along the loop?
This simple pendulum consists of a small bob of mass $m$ suspended by a massless cord of length $l$. The bob is released (without a push) at $t = 0$, where the cord makes an angle $\theta = \theta_0$ to the vertical. At the lowest point of the swing, find the tension in the cord, Ignore friction and air resistance.
A ball of mass \( m = 2.60 \text{ kg} \), starting from rest, falls a vertical distance \( h = 55.0 \text{ cm} \) before striking a vertical coiled spring, which it compresses an amount \( Y = 15.0 \text{ cm} \). Determine the spring stiffness constant of the spring. Assume the spring has negligible mass, and ignore air resistance. Measure all distances from the point where the ball first touches the uncompressed spring \( (y = 0 \text{ at this point}) \).
An object of mass 0.5 kg starts from rest and slides a distance $d$ down a frictionless incline of angle $30^\circ$. While sliding, it contacts an unstressed spring of negligible mass and the object slides an additional distance $10 \text{ cm}$ as it is brought momentarily to rest by compression of the spring (of force constant $200 \text{ N/m}$). Find the initial separation $d$ between object and spring.
Effect of Nonconservative forces

Nonconservative, or dissipative, forces:
Friction
Heat
Electrical energy
Chemical energy
and more

do not conserve mechanical energy. However, when these forces are taken into account, the total energy is still conserved:

\[ \Delta E_{\text{mech}} = \Delta K + \Delta U = W_{\text{nc}} \]

or \[ \Delta K + \Delta U + [\text{change in all other forms of energy}] = 0. \]
The roller-coaster car shown reaches a vertical height of only 25 m on the second hill before coming to a momentary stop. It traveled a total distance of 400 m.

Determine the thermal energy produced and estimate the average friction force (assume it is roughly constant) on the car, whose mass is 1000 kg.
A 50.0-kg block and 100-kg block are connected by a string. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50-kg block and incline is 0.250. Determine the change in the kinetic energy of the 50-kg block as it moves from A to B, a distance of 20.0 m.
• A 10.0-kg block is released from point A. The track is frictionless except for the portion between points B and C, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2000 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between B and C.
Far from the surface of the Earth, the force of gravity is not constant:

\[
\vec{F} = -G \frac{mM_E}{r^2} \hat{r}.
\]

The work done on an object moving in the Earth’s gravitational field is given by:

\[
W = \int_{1}^{2} \vec{F} \cdot d\vec{l} = -GmM_E \int_{1}^{2} \frac{\hat{r} \cdot d\vec{l}}{r^2}.
\]
Solving the integral gives:

\[ W = \frac{GmM_E}{r_2} - \frac{GmM_E}{r_1}. \]

Because the value of the integral depends only on the end points, the gravitational force is conservative and we can define gravitational potential energy:

\[ U(r) = -\frac{GmM_E}{r}. \]
Escape Velocity

If an object’s initial kinetic energy is equal to the potential energy at the Earth’s surface, its total energy will be zero. The velocity at which this is true is called the escape velocity; for Earth:

\[ v_{esc} = \sqrt{\frac{2GM_E}{r_E}} = 1.12 \times 10^4 \text{ m/s}. \]
A box is allowed to fall from a rocket traveling outward from Earth at a speed of 1800 m/s when 1600 km above the Earth’s surface. The package eventually falls to the Earth. Estimate its speed just before impact. Ignore air resistance.
8-2 Potential Energy and Force

In one dimension,

\[ U(x) = -\int F(x) \, dx + C. \]

We can invert this equation to find \( U(x) \) if we know \( F(x) \):

\[ F(x) = -\frac{dU(x)}{dx}. \]

In three dimensions:

\[ \vec{F}(x, y, z) = -\hat{i} \frac{\partial U}{\partial x} - \hat{j} \frac{\partial U}{\partial y} - \hat{k} \frac{\partial U}{\partial z}. \]
Conservative Forces and Potential Energy – Check

• Look at the case of a deformed spring

\[ F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx \]

– This is Hooke’s Law
Energy Diagrams and Equilibrium

• Motion in a system can be observed in terms of a graph of its position and energy
• In a spring-mass system example, the block oscillates between the turning points, $x = \pm x_{\text{max}}$
• The block will always accelerate back toward $x = 0$
Potential Energy Curve of a Molecule

There is potential energy associated with the force between two neutral atoms in a molecule which can be modeled by the Lennard-Jones function

\[ U(x) = 4\epsilon \left[ \left( \frac{\sigma}{x} \right)^{12} - \left( \frac{\sigma}{x} \right)^{6} \right] \]

- The force is repulsive (positive) at small separations
- The force is zero at the point of stable equilibrium
- The force is attractive (negative) when the separation increases
- At great distances, the force approaches zero
This is a potential energy diagram for a particle moving under the influence of a conservative force. Its behavior will be determined by its total energy.

With energy $E_1$, the object oscillates between $x_3$ and $x_2$, called turning points. An object with energy $E_2$ has four turning points; an object with energy $E_0$ is in stable equilibrium. An object at $x_4$ is in unstable equilibrium.
A particle moves along a line where the potential energy of its system depends on its position \( r \) as graphed in Figure. In the limit as \( r \) increases without bound, \( U(r) \) approaches +1 J.

(a) Identify each equilibrium position for this particle. Indicate whether each is a point of stable, unstable or neutral equilibrium.

(b) The particle will be bound if the total energy of the system is in what range?

(c) Suppose that the system has energy -3 J. Determine the range of positions where the particle can be found.
Power Generalized

• Power can be related to any type of energy transfer
• In general, power can be expressed as

\[ P = \frac{dE}{dt} \]

• \( \frac{dE}{dt} \) is the rate of energy transfer.
Units of Power

• The SI unit of power is called the watt
  – 1 watt = 1 joule / second = 1 kg \cdot m^2 / s^2

• A unit of power in the US Customary system is horsepower
  – 1 hp = 746 W

• Units of power can also be used to express units of work or energy
  – 1 kWh = (1000 W)(3600 s) = 3.6 \times 10^6 J
A 800-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 2.00 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?