Directions: You must answer all questions. To receive any partial credit, you must show your work. Results from the text need not be reproduced in detail - you can merely cite the source.

(10) 1. Are either of the following a HPD region? Why or why not?

(20) 2. Let \( Y \) have pdf \( f(y) = \frac{2(\theta - y)}{\theta^2}, 0 < y < \theta \). Is \( Q = Y/\theta \) a pivotal quantity?

(20) 3. Consider a random sample of size \( T \) from a \( N(\theta, \sigma^2) \) distribution, \( \sigma^2 \) known, which yields the .95 confidence interval \( C(8 < \theta < 12) \). Carefully interpret this interval.
4. Consider Example 6.10.1 on p. 344, where \( Y \) is a Bernoulli random variable with p.m.f. 
\[
\text{Prob}(Y = \theta - 1 | \theta) = \text{Prob}(Y = \theta + 1 | \theta) = .50 \ \text{and} \ \theta \in \mathbb{R}.
\]
Suppose the prior distribution is \( \theta \sim N(0, 1) \) with prior density \( \phi(\cdot) \). Given one observation \( Y = y \), find the posterior distribution of \( \theta \).

5. Let \( Y \) be a random variable with p.m.f. under the simple null hypotheses \( H_1 \) and the simple alternative hypothesis \( H_2 \) given in the following table.

<table>
<thead>
<tr>
<th>( y )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(y</td>
<td>H_1) )</td>
<td>.01</td>
<td>.01</td>
<td>.01</td>
<td>.01</td>
<td>.01</td>
<td>.01</td>
</tr>
<tr>
<td>( f(y</td>
<td>H_2) )</td>
<td>.06</td>
<td>.05</td>
<td>.04</td>
<td>.03</td>
<td>.02</td>
<td>.01</td>
</tr>
</tbody>
</table>

(a) Find the most powerful test for \( H_1 \) versus \( H_2 \) with size .04.
(b) Compute the probability of a Type II error for this test.