MIDTERM EXAM

ECON 220B
Statistics and Econometrics II
(open book)

Directions: You must answer all questions. To receive any partial credit, you must show your work. Results from the text need not be reproduced in detail - you can merely cite the source.

1. Let \( Y_t \) (\( t = 1, 2, \ldots, T \)) be i.i.d. \( N(0, \sigma^2) \), and let \( \theta \) have a \( N(\mu, h^{-1}) \) prior where \( \sigma^2, \mu, \) and \( h \) are all known. In particular, suppose \( \mu = 0, h = .44h, h = T/\sigma^2 = 1, \) and \( \overline{y} = 1.44 \).

(9) (a) Give the .95 HPD interval for \( \sigma^2 \).

(10) (b) Suppose \( \delta = .50 \). What is the ex ante coverage probability of your interval in (a)?

2. Let \( Y_t \) (\( t = 1, 2, \ldots, 5 \)) be a random sample from an exponential distribution with mean \( \beta \) [i.e., EXP(\( \beta \))]. Consider testing \( H_1: \beta = 2 \) versus \( H_2: \beta \neq 2 \). Suppose \( \overline{y} = 1 \).

(15) (a) Construct a Wald test statistic using the information matrix under \( H_1 \). Perform the test at the .05 level and clearly state your conclusion.

(15) (b) Construct the LR test statistic. Perform the test at the .05 level and clearly state your conclusion.

(15) (c) Construct a score test statistic using the information matrix. Perform the test at the .05 level and clearly state your conclusion.

3. Suppose \( Y|\theta \sim N(\theta, 1) \) and \( f(\theta) \) is uniform over \( \mathcal{R} \). Let \( \alpha = \theta^2 \) be the quantity of interest.

(6) (a) Find the posterior distribution of \( \theta \).

(6) (b) Find the posterior distribution of \( \alpha \).

(6) (c) Under quadratic loss, what is the optimal Bayesian point estimate \( \hat{\alpha} \) of \( \alpha \)? Be specific!

(6) (d) Find the risk function \( R(\hat{\alpha} | \alpha) \) for the corresponding Bayesian estimator in (c).

(6) (e) Let \( \mathcal{x} = y^2 - 1 \). Find \( R(\mathcal{x} | \alpha) \).

(6) (f) What do you conclude from (d) and (e)?