Directions: You must answer all of the following questions. To receive partial credit, you must show your work.

1. Consider the standard multiple linear normal regression model with fixed regressors and the conjugate normal gamma prior density for $\beta$ and $\sigma^{-2}$ with hyperparameters $\bar{\beta}$, $A$, $s^{-2}$, and $\gamma > 2$. Answer each of the following questions, fully justifying your answer in each case.

   (6) (a) Are $\beta$ and $\sigma^{-2}$ independent?
   (6) (b) Is $\beta$ mean-independent of $\sigma^{-2}$?
   (6) (c) Are $\beta$ and $\sigma^{-2}$ uncorrelated?

2. Consider the standard multiple linear regression model $y = X\beta + u$ with OLS estimator $\hat{\beta} = (X'X)^{-1}X'y$. Let $\hat{\beta} = Cy$, where $C$ is a fixed $K \times T$ matrix. Show: $\hat{\beta}$ is an unbiased estimator of $\beta$ if and only if $\text{Cov}(\hat{\beta}, b) = \text{Var}(b)$.

3. The following questions refer to your computer homework assignment. You must hand in your computer output in order to receive any credit for your answers.

   (5) (a) Consider regression 1(e). Test the null hypothesis all slopes are zero at the .01 level of significance.
   (10) (b) Consider regression 1(b). Test the null hypothesis that labour market experience has no effect at the .01 level of significance.
   (10) (c) Consider regression 1(b). Test $\beta_{14} = \beta_{15}$ versus $\beta_{14} > \beta_{15}$ at the .05 level of significance.
4. Consider the normal linear regression model

\[ y = X\beta + u = x\beta_1 + x\beta_2 + u, \]

where \( K = 2 \), \( \beta = [\beta_1, \beta_2] \), \( X = [x, x] \), and \( x \) is a \( T \times 1 \) vector. Clearly, there exists perfect multicollinearity since both columns of \( X \) are identical. Suppose your prior information is given by the normal-gamma distribution \( \text{NG}(b, Q, s^{-2}, \psi) \), where \( Q = A^{-1} \), \( b = [b_1, 0]' \), \( s^{-2} \), \( \psi \), and

\[
A = \begin{bmatrix}
a_1 & 0 \\
0 & a_2
\end{bmatrix}
\]

are known.

(a) Find \( E(\beta \mid y, \sigma) \) as \( a_1 \to 0 \).

(b) Find \( \text{Var}(\beta \mid y, \sigma) \) as \( a_1 \to 0 \).

(c) In the face of the perfect collinearity and the partially noninformative prior as \( a_1 \to 0 \), is it possible to draw inferences about \( \beta_1 \) and \( \beta_2 \) \emph{separately}? If so, why? If not, why not?

(d) Suppose you are interested in \( \beta_2 \). Is multicollinearity a problem? Consider both the case \( a_1 > 0 \) and the case as \( a_1 \to 0 \).