FINAL EXAM

ECON 220B
Statistics and Econometrics II
(open book)

**Directions**: You must answer all of the following questions. To receive partial credit, you must show your work.

1. Suppose \( Y \sim \text{N}(\theta, 1) \) and you have a single observation \( Y = y \).

   (10) (a) Suppose \( \theta \in \mathbb{R} \). Construct a .95 confidence interval for \( \theta \).

   (10) (b) Suppose \( \theta \in \mathbb{R}^+ \). Show that the interval

   \[
   \max(\{y - 1.96, 0\}) < \theta < \max(\{y + 1.96, 0\})
   \]

   is a .95 confidence interval for \( \theta \).

   (10) (c) Suppose you observe \( y = -2 \) in (b). Interpret your interval.

2. Consider the generalized regression model \( y = X\beta + u, \ E(u) = 0, \ E(uu') = \sigma^2\Omega \) with fixed regressors and some positive definite matrix \( \Omega \). Let \( \hat{\beta} \) and \( \hat{\beta}_{\text{GLS}} \) denote the OLS and GLS estimators, respectively. Define the OLS and GLS residuals \( \hat{u} = y - X\hat{\beta} \) and \( \hat{u}_{\text{GLS}} = y - X\hat{\beta}_{\text{GLS}} \), respectively. Let \( \hat{\beta} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y \) and \( \hat{u} = y - X\hat{\beta} \) for some positive definite matrix \( \Omega \).

   (5) (a) Find \( \text{Var}(\hat{\beta}) \).

   (5) (b) Find \( \text{Cov}(b, \hat{\beta}) \).

   (5) (c) Find \( \text{Cov}(\hat{\beta}_{\text{GLS}}, \hat{\beta}) \).

   (5) (d) Find \( \text{Var}(\hat{u}) \).

   (5) (e) Find \( \text{Cov}(\hat{u}_{\text{GLS}}, \hat{u}) \).

   (5) (f) Find \( \text{Cov}(\hat{u}, \hat{u}) \).
3. The following refer to the computer homework assignment.

(10) (a) Consider regression 1(c). Let $b_4$ and $b_{16}$ be the coefficients of $x_4$ and $x_{16}$ respectively. Using only your output from regression 1(c), test the null hypothesis that $b_4 = b_{16} = 0$ at 0.05 level of significance. You must hand in your computer output in order to receive any credit for your answers.

(20) (b) Consider Question III, part 1. Assuming quadratic loss, find the Bayesian point estimator for $\theta$.

(10) (c) Question III, part 3. Hand in your computer output.