Prob: Given function $f(x)$, estimate the area under the graph of $f(x)$

e.g. 1 (Sample Midterm 1, 1, a)

Estimate the area under the graph of $f(x) = x^2 + x$ from $x = 0$ to $x = 3$ using 3 approximating rectangles and left endpoints

\[ f(x) = x^2 + x \]

\[ f(0) = 0, \quad f(1) = 1 + 1 = 2, \quad f(2) = 2^2 + 2 = 6 \]

Step 2: find width of subintervals
\[ \Delta x = \frac{3 - 0}{3} = 1 \]

This gives you the three left endpoints 0, 1, 2

Step 3: finish the graph (with 3 rectangles)

Note that the 1st rectangle vanishes

Step 4: plug into the formula of $L_3$

\[ L_3 = 1 \cdot 0 + 1 \cdot 2 + 1 \cdot 6 = 8 \]
Rmk: in Hw 1 & 2, the height of the rectangles is reading directly from the given graph of f.

\[ \sum_{i=m}^{n} A_i = A_m + A_{m+1} + \ldots + A_n \]

Appendix E: Sigma notation (Page A34 - A38)

Definition: Given m, n two integers, m \( \leq \) n, and \( a_m, a_{m+1}, \ldots, a_n \) are real numbers, then

Rmk: in most cases, we will only meet \( m=0 \) or \( m=1 \), which means we get a sequence of numbers

\[ a_0, a_1, a_2, \ldots, a_n \]

\[ \sum_{i=0}^{n} A_i = a_0 + a_1 + \ldots + a_n \quad , \quad \sum_{i=1}^{n} A_i = a_1 + a_2 + \ldots + a_n \]

Examples: 0: \[ \sum_{i=1}^{4} i = 1 + 2 + 3 + 4 \]

1: \[ \sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2 = \sum_{i=1}^{4} i^2 = \sum_{j=1}^{4} j^2 \] (the index can be changed)
3. (Sum of constants)
\[ \sum_{i=1}^{5} 3 = 3 + 3 + 3 + 3 + 3 \]
(totally 5 "3")

4. Multiplicative constant can be pulled out.
\[ \sum_{i=1}^{n} C \cdot a_i = C \sum_{i=1}^{n} a_i \quad C \text{ is a constant} \]

\[ \sum_{i=1}^{n} (C + a_i) \neq C + \sum_{i=1}^{n} a_i \quad \text{(wrong!)} \]

\[ \sum_{i=1}^{3} \frac{i^4}{6} = \frac{1^4}{6} + \frac{2^4}{6} + \frac{3^4}{6} = \frac{1^4 + 2^4 + 3^4}{6} = \frac{1}{6} (1^4 + 2^4 + 3^4) \]
\[ = \frac{1}{6} \sum_{i=1}^{3} i^4 \]

5. \[ \sum_{i=1}^{n} (a_i + bi) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} bi \]

\[ \sum_{i=1}^{5} (i + i^3) = (1+1^3) + (2+2^3) + (3+3^3) + (4+4^3) + (5+5^3) = \sum_{i=1}^{5} i + \sum_{i=1}^{5} i^3 \]

\[ \frac{\sum_{i=1}^{n} i}{2} = \frac{n}{2} \binom{n+1}{2} \quad \frac{\sum_{i=1}^{5} i}{2} = 1+2+3+4+5 = \frac{4 \cdot (4+1)}{2} \]

\[ \frac{\sum_{i=1}^{n} i^2}{6} = \frac{n(n+1)(2n+1)}{6} \]

\[ \frac{\sum_{i=1}^{5} i^2}{6} = \frac{5(5+1)(2 \cdot 5+1)}{6} \]

\[ \frac{\sum_{i=1}^{5} i^3}{6} = \left( \frac{\sum_{i=1}^{n} i}{2} \right)^2 \]