Practice for testing convergence/divergence of sequence/series 11.1-11.2, 44350

(16 points, 4 points each) For each given $a_n$, determine whether the sequence $\{a_n\}_{n=1}^{\infty}$ and the corresponding series $\sum_{n=1}^{\infty} a_n$ are convergent or divergent. Scratch the reason.

1. $a_n = \frac{2n}{3n - 1}$

   The sequence $\{a_n\}_{n=1}^{\infty}$ is __________.

   The series $\sum_{n=1}^{\infty} a_n$ is __________.

   Reason (Scratch work):

2. $a_n = e^{-\frac{n}{2}}$

   The sequence $\{a_n\}_{n=1}^{\infty}$ is __________.

   The series $\sum_{n=1}^{\infty} a_n$ is __________.

   Reason (Scratch work):

3. $a_n = e^{\frac{n}{2}} - e^{\frac{2}{n+1}}$

   The sequence $\{a_n\}_{n=1}^{\infty}$ is __________.

   The series $\sum_{n=1}^{\infty} a_n$ is __________.

   Reason (Scratch work):

4. $a_n = n^{-\frac{2}{e}}$

   The sequence $\{a_n\}_{n=1}^{\infty}$ is __________.

   The series $\sum_{n=1}^{\infty} a_n$ is __________.

   Reason (Scratch work):
(Remark: For the "Scratch Work", you do not need to write in such detail.)

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(16 points, 4 points each) For each given \( a_n \), determine whether the sequence \( \{a_n\}_{n=1}^{\infty} \) and the corresponding series \( \sum_{n=1}^{\infty} a_n \) are convergent or divergent. Scratch the reason.

1. \( a_n = \frac{2n}{3n - 1} \)
   
   The sequence \( \{a_n\}_{n=1}^{\infty} \) is \textbf{convergent}.
   
   The series \( \sum_{n=1}^{\infty} a_n \) is \textbf{divergent}.
   
   **Reason (Scratch work):**
   \[
   \lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{2n}{3n - 1} = \lim_{n \to \infty} \frac{2}{3} = \frac{2}{3}.
   \]
   
   Therefore, \( a_n \) is convergent.
   
   However, notice that \( \lim_{n \to \infty} a_n \neq 0 \), according to TEST FOR DIVERGENCE, the series \( \sum_{n=1}^{\infty} a_n \) is divergent.

2. \( a_n = e^{-\frac{n}{2}} \)
   
   The sequence \( \{a_n\}_{n=1}^{\infty} \) is \textbf{convergent}.
   
   The series \( \sum_{n=1}^{\infty} a_n \) is \textbf{convergent}.
   
   **Reason (Scratch work):**
   \[
   \lim_{n \to \infty} e^{-\frac{n}{2}} = e^{-\frac{\infty}{2}} = 0, \text{ i.e., } a_n \text{ is convergent.}
   \]

3. \( a_n = e^{\frac{n}{2}} - e^{\frac{n}{3}} \)
   
   The sequence \( \{a_n\}_{n=1}^{\infty} \) is \textbf{convergent}.
   
   The series \( \sum_{n=1}^{\infty} a_n \) is \textbf{convergent}.
   
   **Reason (Scratch work):**
   \[
   \lim_{n \to \infty} e^{\frac{n}{2}} - e^{\frac{n}{3}} = e^{\frac{\infty}{2}} - e^{\frac{\infty}{3}} = 0 - 0 = 0,
   \]
   
   therefore, \( a_n \) is convergent.

4. \( a_n = n^{-\frac{3}{2}} \)
   
   The sequence \( \{a_n\}_{n=1}^{\infty} \) is \textbf{convergent}.
   
   The series \( \sum_{n=1}^{\infty} a_n \) is \textbf{divergent}.
   
   **Reason (Scratch work):**
   \[
   a_n = n^{-\frac{3}{2}} = \frac{1}{n^{\frac{3}{2}}}, \lim_{n \to \infty} a_n = \frac{1}{\infty} = 0
   \]
   
   \( a_n \) is convergent.

The partial sum \( S_n = a_1 + a_2 + \ldots + a_n \)

\[
= (e^\frac{2}{1} - e^\frac{2}{3}) + (e^\frac{2}{3} - e^\frac{2}{5}) + (e^\frac{2}{5} - e^\frac{2}{7}) + \ldots
\]

\[
= e^\frac{2}{1} - e^\frac{2}{n+1}
\]

Therefore, \[
\frac{1}{n+1} \sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} S_n = e^2 - \lim_{n \to \infty} e^\frac{2}{n+1} = e^2 - 0, \text{ ConV.}
\]