2015 Fall Math 2B (44351)  Quiz 4 (10/30)

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. No notes, phones and calculators. You have 25 minutes to take this 10 point quiz.

Please also put your name in the back of the quiz.

Student Name:  

1. (2 pts) Evaluate the indefinite integral.

\[
\int \frac{dx}{x^2 + 3} = \int \frac{\sqrt{3} \sec^2 \theta \, d\theta}{3(\tan^2 \theta + 1)} = \frac{1}{\sqrt{3}} \int \sec^2 \theta \, d\theta = \frac{1}{\sqrt{3}} \tan \theta + C = \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) + C
\]

2. (4 pts) Evaluate the indefinite integral

\[
\int \frac{dt}{t^2 \sqrt{t^2 - 4}} \quad \text{let} \quad t = 2 \sec \theta , \quad dt = 2 \sec \theta \tan \theta \, d\theta \quad \frac{\sqrt{t^2 - 4}}{2} \quad \frac{\sqrt{t^2 - 4}}{2} + C
\]

\[
\int \frac{dt}{t^2 \sqrt{t^2 - 4}} = \int \frac{2 \sec \theta \tan \theta}{4 \sec^2 \theta \cdot 2 \tan \theta} \, d\theta = \frac{1}{4} \int \frac{1}{\sec \theta} \, d\theta = \frac{1}{4} \int \cos \theta \, d\theta = \frac{1}{4} \sin \theta + C = \frac{\sqrt{t^2 - 4}}{4t} + C
\]

3. (4 pts) Evaluate the definite integral

\[
\int_0^1 \frac{x - 4}{x^2 - 5x + 6} \, dx
\]

\[
\frac{x - 4}{x^2 - 5x + 6} = \frac{x - 4}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3) + B(x-2)}{(x-2)(x-3)}
\]

\[
x - 4 = A(x - 3) + B(x - 2) \quad x = 3 \Rightarrow -1 = B
\]

\[
x = 2 \Rightarrow -2 = -A , \quad A = 2
\]

\[
\int_0^1 \frac{x - 4}{x^2 - 5x + 6} \, dx = \int_0^1 \frac{2}{x-2} - \frac{1}{x-3} \, dx = \left. \left( 2 \ln |x-2| - \ln |x-3| \right) \right|_0^1 = -\ln 2 - 2 \ln 2 + \ln 3 = -3 \ln 2 + \ln 3
\]